Interaction between neutrino flavor oscillation and Dark Energy as a super-luminal propagation

Marco Lelli

As it is well known a recent series of experiments, conducted in collaboration between CERN laboratories in Geneva and the Gran Sasso National Laboratory for Particle Physics, could have decreed the discovery of the transmission of a beam of super-luminal particles.

Experimental data indicate that the distance between two laboratories (approximately 730 km) was covered by a beam of neutrinos with an advance of approx 60 nanoseconds with respect to a signal travelling at the relativistic limit speed c (which takes a time interval of the order of 2,4.10-3 s to perform the way).

Neutrino beam starts from CERN and after travelling 730 km through the Earth's crust, affects lead atoms of the OPERA detector at Gran Sasso laboratories. Production of neutrino beam is due by the acceleration and collision of protons and heavy nuclei. This event produces pions and kaons, which then decay into muons and ν_μ.

The initial energy of neutrino beam is 17 GeV and its composition is almost entirely due to ν_μ.

Publication of the OPERA experimental data immediately got a deep world mass-media echoes: the possible confirmation of the results of the experiment seems to imply an explanation leading to change our current thoughts about theory of relativity and, therefore, the intimate space-time nature. In this assumption c may not be considered a speed limit on the quantum scale investigation.

In this paper we try to show how the uncertainty principle and the oscillation in flavor eingenstates of neutrino beam may provide a possible explanation for OPERA's data.

Our research assumes two basic hypotheses.

First approximation: approximation in number of flavor eigenstates (and then in mass eigenstates) within is supposed to play neutrino oscillation.

We consider this oscillation between two flavor eigenstates. Then we assume that each component of the neutrino beam can be described by a linear combination of two eigenstates of flavor. These two eigenstates are: μ flavor (the flavor of neutrino beam generation) and τ flavor. Oscillations in this two flavor was already observed in first half of 2010 within the same OPERA experimental series .

Although, as it is known, the neutrino oscillation cover three mass eigenstates for its complete description, we assume here an approximation for dominant mass of neutrino τ, which reduces the description of neutrino propagation in a linear combination of only two mass eigenstates.

In this approximation we can now describe the propagation of each neutrino produced at CERN as a combination of two mass eigenstates as follows:

$$|\nu(t)\rangle = \sum_i P_i \; |\nu_i(t)\rangle \qquad (1)$$

$i = 1,2$

Flavor and mass eigenstates are related by a unitary transformation which implies a mixing angle in vacuum similar to Cabibbo mixing angle for flavor of quarks:

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} cos\theta & sen\theta \\ -sen\theta & cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad (2)$$

then

$$|\nu_\mu\rangle = \cos\theta|\nu_1\rangle + \mathrm{sen}\theta|\nu_2\rangle$$
$$|\nu_\tau\rangle = -\mathrm{sen}|\nu_1\rangle + \cos\theta|\nu_2\rangle \quad (3)$$

Second approximation: we suppose that propagation of neutrino beam is in vacuum. The propagation in vacuum is determined by the temporal evolution of the mass eigenstates

$$|\nu_1(t)\rangle = e^{-i\frac{E_1 t}{h}}|\nu_1(0)\rangle$$
$$|\nu_2(t)\rangle = e^{-i\frac{E_2 t}{h}}|\nu_2(0)\rangle \quad (4)$$

We can consider valid this assumption, at least in first approximation, because matter interacts in particular with ν_e and less with ν_μ and ν_τ. ν_e weakly interacts with matter by $W^\pm$ and Z^0 bosons while ν_μ and ν_τ only by Z^0 bosons. So the principal possible effect consists in a massive transformation of ν_e in the $|\nu_2\rangle$ eigenstate.

Considering the small number of ν_e in starting beam we can neglect this effect.

Assuming that in the initial state only ν_μ are present in the beam, through a series of elementary steps, we can get

$$|\nu_\mu(t)\rangle = \cos^2\theta e^{-i\frac{E_1 t}{h}}|\nu_1(0)\rangle$$
$$+ \mathrm{sen}^2\theta e^{-i\frac{E_2 t}{h}}|\nu_2(0)\rangle \quad (5)$$

then we can obtain the probability

$$\langle\nu_\mu(t)|\nu_\mu(t)\rangle$$
$$= I_\mu^0\left(1 - \mathrm{sen}^2 2\theta\, \mathrm{sen}^2\left(\frac{(E_2 - E_1)t}{2h}\right)\right) \quad (6)$$
$$I_\mu^0 = \langle\nu_\mu(0)|\nu_\mu(0)\rangle$$

In the approximation $m_\mu \ll E_\mu$ we can write

$$(E_2 - E_1) \cong \Delta m_{12}^2/2E$$

and finally the transition probabilities between eigenstates of flavor

$$P(\nu_\mu \to \nu_\mu) = 1 - \mathrm{sen}^2 2\theta_{12}\, \mathrm{sen}^2\left(\frac{\Delta m_{12}^2 t}{4hE}\right)$$

$$P(\nu_\mu \to \nu_\tau) = \mathrm{sen}^2 2\theta_{12}\, \mathrm{sen}^2\left(\frac{\Delta m_{12}^2 t}{4hE}\right) \quad (7)$$

ν_μ beam produced at CERN propagates as a linear superposition of mass eingestates given by the following relation

$$|\nu(t)\rangle = P(\nu_\mu \to \nu_\mu)|\nu_1(t)\rangle$$
$$+ P(\nu_\mu \to \nu_\tau)|\nu_2(t)\rangle \quad (8)$$

This superposition generates an uncertainty in propagating mass neutrino that grows over time and is equal to

$$\Delta|\nu(t)\rangle = |\nu(t)\rangle - |\nu(0)\rangle \quad (9)$$
$$\Delta m_\nu =$$
$$m_1 - \left(\langle\nu_\mu(t)|\nu_\mu(t)\rangle m_1 + \langle\nu_\mu(t)|\nu_\tau(t)\rangle m_2\right) \quad (10)$$

This uncertainty in the mass eigenstates of the neutrino implies an uncertainty in the energy of propagation.

Given the relativistic equation

$$E^2 = p^2c^2 + m_0^2c^4$$

taking the momentum of propagation $\vec{p} = cost$, the uncertainty linked to neutrino mass eigenstate is linearly reflected in an uncertainty in the propagation energy:

$$\Delta E = \Delta m_0 c^2 \quad (11)$$

Therefore we have

$$\Delta E = \Delta m_\nu =$$
$$m_1 - \left(\langle\nu_\mu(t)|\nu_\mu(t)\rangle m_1 + \langle\nu_\mu(t)|\nu_\tau(t)\rangle m_2\right) =$$
$$= m_1 - \left(\left(1 - \langle\nu_\mu(t)|\nu_\tau(t)\rangle\right)m_1\right.$$
$$\left. + \langle\nu_\mu(t)|\nu_\tau(t)\rangle m_2\right)$$
$$= \langle\nu_\mu(t)|\nu_\tau(t)\rangle \Delta m_{12} \quad (12)$$

Following the uncertainty principle we have

$$\Delta E \Delta t \geq \frac{\hbar}{2} \approx \hbar$$

so the uncertainty (12), about the value of ν_μ energy of propagation, causes a corresponding uncertainty in its time of flight between the point of production and the point of arrival.

This uncertainty is expressed as follows:

$$\Delta t \ \geq \ \frac{\hbar}{2\Delta E} \geq \frac{\hbar}{2\langle \nu_\mu(t) | \nu_\tau(t) \rangle \Delta m_{12}} \ \approx \frac{\hbar}{\langle \nu_\mu(t) | \nu_\tau(t) \rangle \Delta m_{12}} \qquad (13)$$

In OPERA case available experimental data are:

t = 2,4 10-3 s
E=17 GeV

Assuming $sen^2 2\theta_{12} = 1$, in analogy with the value attributed to Cabibbo quark mixing angles, and a value for $\Delta m_{12} \approx 10^{-2} eV \approx 1{,}6.\,10^{-21}$J we have

$$\langle \nu_\mu(t) | \nu_\tau(t) \rangle \ \approx 7{,}3\ .\ 10^{-7}$$

$$\Delta E = \langle \nu_\mu(t) | \nu_\tau(t) \rangle \Delta m_{12} \approx 7{,}3\ .10^{-7}.\ 1{,}6\ .\ 10^{-21} \approx 1{,}16\ .\ 10^{-27}\text{J}$$

then

$$\Delta t \ \geq \frac{\hbar}{2\Delta E} \approx \frac{\hbar}{\Delta E} \geq \ 4{,}5.\,10^{-8}\ s \ \ \approx 9.\,10^{-8}\ s \qquad (14)$$

(14) shows that the advance Δt on the propagation of neutrino beam, detected in the execution OPERA experiment, is between the range determined by the uncertainty principle.

The advance Δt is then interpreted by the uncertainty principle and the neutrino flavor oscillation during propagation. This oscillation implies an uncertainty in the neutrino propagation energy, due to the linear superposition of its mass eigenstates, which affects the uncertainty of its flight time.

According to this interpretation, therefore, the results of OPERA experiment, if confirmed, would represent not a refusal of the condition of c as a relativistic speed limit, but rather a stunning example of neutrino flavor oscillation according to physics's laws known today (uncertainty principle and speed limit c).

The range indicated in (14) depends on the competition of two factors. On one hand, the intrinsic nature of inequality of the uncertainty principle, on the other our fuzzy knowledge of Δm_{12} between mass eigenstates of neutrinos with different flavors.

One of the most convincing experimental proofs of flavor neutrino oscillation is the lack of solar electron neutrinos measured experimentally respect to the theoretically expected flow.

OPERA, as well as other tests, was designed to observe possible flavor oscillation in a neutrino beam running along the earth's subsurface. Any oscillation can be found by observing a change of flavor in a fraction of neutrinos in the arrive.

However, if this happens, neutrino mass eigenstate is described by a linear superposition of mass eigenstates of pure muon neutrino and tau neutrino.

This condition generates an uncertainty on the propagation energy, which translates into an uncertainty on the flight time.

This is directly proportional to the total flight time and the square of the difference between the mass values of the different flavors of neutrinos, while it is inversely proportional to the total energy of the beam.

In this interpretation, therefore, the advance of the flight time of the neutrino beam with respect to the velocity c, far from being a refutation of the relativistic speed limit, is a good demonstration of neutrino flavor oscillation.

So we could use the advantage Δt in an attempt to determine, more accurately, the value of Δm_{12}.

On the other hand, examples of physical effects equivalent to a super-luminal propagation of particles are considered in other fields of contemporary theoretical physics. Hawking effect about the emission temperature of a Black Hole is, under this respect, a very significant example.

Cosmic neutrinos flavor oscillations. We can now consider what could be the value of the advantage Δt respect to the time of flight of c in the case of neutrinos coming, for example, from a SuperNova explosion.

In this case the average energy of neutrinos ν_e is of the order of 10^7 eV and the time of flight, for example in the case of SuperNova 1987a, of the order of 10^{12} s.

Under these conditions we have

$$\langle \nu_\mu(t) | \nu_\tau(t) \rangle \to 1$$

and it is conceivable that it may start a continuous sequence of oscillations in mass eigenstates.

The logical consequence of this situation is a superposition of two equally probable mass eigenstates.

$$|\nu(t)\rangle = \frac{1}{2}|\nu_1(t)\rangle + \frac{1}{2}|\nu_2(t)\rangle \qquad (15)$$

We lose the information of to the initial state of the emitted neutrino along the way.

So the uncertainty in mass eigenstates exists with respect to the state of arrival of the neutrino and a mixing of mass eigenstates with the same probability equal to $\frac{1}{2}$.

In this hypothesis we have

$$\Delta E = \frac{1}{2}\Delta m_{12} \approx 10^{-21}\mathrm{J}$$

$$\Delta t \geq \frac{\hbar}{2\Delta E} \approx \frac{\hbar}{\Delta E} \approx 10^{-14}\, s \qquad (16)$$

therefore an advantage Δt of approx six orders of magnitude lower than in the OPERA case.

Interpretation of the principle of uncertainty used above. The uncertainty principle is commonly intended as an aid to explanation for the impossibility of determining, by observation, contemporarily the position and momentum of a physical system, with absolute precision, because the one excludes the other.

Assuming this interpretation the uncertainty principle could explain , in the case of OPERA, a set of measures centered on an advance $\Delta t = 0$ with a spread on the obtained measurement results in the order of (14).

In contrast, the experimental measurements provided by OPERA appears to be centered on a value of Δt ≈ 60 ns in advance respect to the time of flight of c!

Which explanation is therefore possible to give to the application of the uncertainty principle to justify the consistency of the data provided by OPERA with the fundamental laws of physics known today?

The most coherent interpretation seems to be as follows: the temporal evolution of the neutrino mass eigenstate introduces a temporal evolution in the state of total energy that interacts with space-time producing a reduction of the time of flight. This interaction has to be coherent with the uncertainty principle.

Energy gained or released by neutrino, during oscillation, must be released or gained by space-time, according to the principle of conservation of energy.

A more accurate explanation will require the introduction of some new hypotheses.

We suppose below that space-time possesses a quantized structure. We define a fundamental 1D string element that has the dimension of a length or a time. This fundamental element is a 1D vector in the 2D string wolrdsheet: we call this element the quantum of space-time.

To each 1D of space-time is associated a 1D energy-momentum vector (the total energy associated to a quantum of space-time) that is related to the module of the 1D quantum of space-time with a relation of constraint that we define below.

To introduce the basic unit of space-time we introduce the Polyakov 2D string action and we proceed to its quantization finding the 1D elementary quantum of space-time

$$S = \frac{T}{2}\int d\sigma\, d\tau \sqrt{-h}\; h^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \qquad (17)$$

Now we want to consider (17) in the limit n → 1. The infinitesimal parameters dσ and dτ take the meaning of physically limit movement along, respectively, the spatial direction and temporal direction of the 2D string worldsheet.

We can call these limit movement as follows

$$d\sigma = \Omega^x \qquad lim \ \mathrm{n} \rightarrow 1$$
$$d\tau = \Omega^0 \qquad lim \ \mathrm{n} \rightarrow 1$$
$$\hat{\imath}\Omega^x = -\hat{\jmath}c\,\Omega^0 \tag{18}$$

Ω^x e Ω^0 take the meaning of quantum of space-time in space direction and time direction in the 2D string worldsheet.

Therefore, in this case, to each spatial direction of the elementary string element corresponds a temporal direction that, in a Minkowski's manifold, is orthogonal to the space direction. The relation (18) binds the module of the element of string along the spatial direction with respect to temporal direction, in the case of a Minkowski's manifold, and have the values ℓ_P and $\frac{\ell_P}{c}$.

Double differentiation $\partial_\alpha X^\mu \partial_\beta X^\nu$ appearing in (17) must now be rewritten taking into account that in a Minkowski's manifold, for relations (18), we can write

$$dx^1, dx^2 ..., dx^n = \Omega^1, \Omega^2 ..., \Omega^n$$

then

$$\partial_\alpha X^\mu = \frac{\partial X^\mu}{\partial x^\alpha} = \frac{(X^\mu + \Omega^\mu) - (X^\mu)}{\Omega^\alpha} = \delta^\mu_\alpha \tag{19}$$

Since it is possible to show that 2D string worldsheet action of Polyakov coincides with Nanbu-Goto action

$$S = \frac{T}{c}\int dA = \frac{T}{c}\int d\tau\, d\sigma \sqrt{-\gamma}$$

given the relation

$$\sqrt{-h}h_{\alpha\beta} = \sqrt{-\gamma}\gamma_{\alpha\beta}$$

and because we have

$$\gamma_{\alpha\beta} = \partial_\alpha X \partial_\beta X = \delta^\alpha_\alpha \delta^\beta_\beta$$

we can rewrite (18) as follows

$$h = T_{\mu 0}\,\Omega^\mu \Omega^0 \tag{20}$$

In (20) with $T_{\mu\nu}$ we have indicated relation $T_{\mu\nu} = T\eta_{\mu\nu}$. So we indicate string tension in 2 dimensions as a tensor of rank 2.

In a Minkowski's manifold we have:

$$\Omega^\mu = \ell_P{}^\mu \qquad \Omega^0 = \frac{\ell_P{}^0}{c}$$
$$T_{\mu 0} = \frac{hc\eta_{\mu 0}}{\ell_P{}^2} = \frac{hc}{\eta^{\mu 0}\ell_P{}^2}$$
$$T_{\mu\nu} = \frac{hc\eta_{\mu\nu}}{\ell_P{}^2} = \frac{hc}{\eta^{\mu\nu}\ell_P{}^2} \tag{21}$$

So the string tension in a Minkowski's manifold can be written as a tensor of rank 2 whose product with the module of the fundamental string elements (the quantum of space-time) in spatial and temporal direction is constant and equal to Planck's quantum of action.

Contracting one of the two indices of tension (21) with one of the two vectors Ω^μ or Ω^ν we get the 2D energy-momentum vector for the string element along the direction μ and ν respectively,

$$E_\nu = \frac{hc\eta_{\mu\nu}}{\ell_P{}^2}\,\Omega^\mu = \frac{h}{\ell_P{}^\nu} \tag{22}$$

it is now possible to define the following relation

$$E_\nu\,\Omega^\nu = h \tag{23}$$

Relation (23) was obtained in a Minkowski's manifold: it is therefore valid in a region of space-time in which the action of gravitational energy is negligible. Under these conditions (23) defines a relation of constraint: the product of the 1D length of the fundamental string element (the length of the module of the quantum of space-time) and the 2D energy-momentum vector of 2D string worldsheet associated with this element is constant and equal to Planck's constant.

2D energy- momentum vector E_ν thus defines the expectation value of energy of empty space that corresponds to the amount of energy needed to increase string length of an element of length ℓ_P along ν direction.

Similarly we can define E_ν as the 2D energy- momentum vector associated with the increase of a quantum of space-time along ν direction. For these reasons, in a Minkowski's manifold, (23) takes the form:

$$\overline{E_\nu}\ \ell_P{}^\nu = h \tag{24}$$

valids in each quantum of space-time.

Calculation of the anticipation Δt *in the time of flight.* (24) can be written taking into account variations in the 2D string worldsheet fundamental element:

$$(E_\nu + \delta E_\nu)\ (\Omega^\nu + \delta\Omega^\nu) = h \tag{25}$$

multiplying the two members is obtained the variational relation of least action for the elementary 2D string worldsheet:

$$\Omega^\nu \delta E_\nu + E_\nu \delta\Omega^\nu + \delta E_\nu \delta\Omega^\nu\ = 0 \tag{26}$$

so we have

$$\delta\Omega^\nu = \frac{h}{(E_\nu + \delta E_\nu)} - \Omega^\nu = -\frac{h\delta E_\nu}{E_\nu(E_\nu + \delta E_\nu)} \approx -\frac{h\delta E_\nu}{(E_\nu)^2} \tag{27}$$

and then

$$\int_{CERN}^{Gran\ Sasso} \delta\Omega^\nu \approx -h\int_\varepsilon^{\Delta E} \frac{\delta E_\nu}{(E_\nu)^2} = k - \frac{h}{\varepsilon} + \frac{h}{\Delta E} = \frac{h}{\langle \nu_\mu(t) | \nu_\tau(t) \rangle \Delta m_{12}} \tag{28}$$

From (28) we obtain (13) and the result (14). In (28) the term k is an appropriate constant of integration that take in to account vacuum fluctuations of energy of magnitude ε for the system under investigation.

Conclusions. Conducing our analysis in 2D we quantize the 2D Polyakov string worldsheet action, obtaining a constraint relation that relates 2D energy -momentum vector and the module of 2D elementary string element (the quantum of space-time).

We have therefore assumed that the neutrino flavor oscillation interacts with the energy associated with each element of the 2D worldsheet string (or the space-time) exchanging energy. This exchange is obeying the law of conservation of energy.

This kind of interaction does not require any hypothesis of fifth force, and may, on the contrary, be assumed of gravitational type, in the sense that the energy due to the neutrino mass eigenstates interacts with the energy of the elementary string element with an easy phase overlapping, just as it is with a gravitational mass.

We can therefore assume that neutrino, through the temporal evolution of its mass eigenstates, exchanges energy with space-time. This exchange causes a change, a contraction in the length of the 2D fundamental string element. Integration of this contractions along the path of neutrino flight produces as a result the observed advantage Δt in the time of the flight.

The energy associated with each elementary quantum of 2D string worldsheet in a Minkowski's manifold corresponds to the energy of empty space-time, ie the vacuum energy of the gravitational field in absence of gravitational source. The target of a forthcoming work will be to show how this vacuum energy is able to produce effects phenomenological equivalent to hypothesis of dark energy and dark matter under certain conditions.

Basing on the assumptions here introduced the same uncertainty principle, from first and irreducible principle of physics, assumes the rank of derived condition through (25) - (28) by a more fundamental principle that is (23).

References

[1] B. M. Pontecorvo, Sov. Phys. Usp., 26 (1983) 1087.
[2] L. Wolfenstein, Phys. Rev. D, 17 (1978) 2369.
[3] S. P. Mikheev e A. Yu. Smirnov, Il Nuovo Cimento C, 9 (1986) 17.

[4] S. Braibant, G.Giacomelli, M. Spurio, Particelle ed interazioni fondamentali, Springer, 2010.
[5] J. N. Bahcall, “Neutrino astrophysics” (Cambridge, 1989); http://www.sns.ias.edu/~jnb
[6] http://www.arcetri.astro.it/science/SNe/sn1987a.jpg
[7] H. A. Bethe e J. R. Wilson, Astrophys. J., 295 (1985) 14
[8] G. Pagliaroli, F. Vissani, M. L. Costantini e A. Ianni, Astropart. Phys., 31 (2009) 163.
[9] V. S. Imshennik e O. G. Ryazhskaya, Astron. Lett., 30 (2004) 14.
[10] W. Baade e F. Zwicky, Proc. Natl. Acad. Sci. U.S.A., 20 (1934) 259.
[11] A.M.Polyakov, Gauge Fields and Strings, Harwood academic publishers, 1987.
[12] Measurement of the neutrino velocity with the OPERA detector in the CNGS beam, arXiv:1109.4897.
[13] F. L. Villante e F. Vissani, Phys. Rev. D, 76 (2007) 125019.
[14] F. L. Villante e F. Vissani, Phys. Rev. D, 78 (2008) 103007.
[16] M. A. Markov, “The Neutrino” (Dubna) 1963