Formulas for Magnetic Moments of the Proton and Neutron

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Based on numerical analysis of experimental data, we find simple phenomenological formulas for the magnetic moments of the proton and neutron with 10 valid digits. We also obtain a compact formula for the relation of the electron's anomalous moment to the summary magnetic moment of the nucleon with 11 valid digits, and propose dependencies of the neutron and proton masses in electron mass units as functions with arguments π and fine-structure constant.

Keywords: proton magnetic moment; neutron magnetic moment; phenomenological formula; proton and neutron masses

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Short Content

Modern ideas presume that the nucleon has a complex structure. Accordingly, one should not expect simple and exact formulas for the above quantities in future theory. Still, if we suppose that future theory possesses hidden symmetry then there possibly exist simple formulas for the magnetic moments, since in quantum theory, symmetries normally generate comparatively simple formulas involving integer numbers. The hypothesis can be verified by a simple, but not at all obvious, method of numerical analysis of the experimental data.

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1. Introduction

The latest experimental data for the proton's and neutron's magnetic moments yield 10 digits [1].

The magnetic moment of the proton in Bohr magneton units is

$$\mu_p = 1.521032210(12)\times 10^{-3}, \quad (1)$$

while that of the neutron

$$\mu_n \cong 1.04187563(25). \qquad (2)$$

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Theoretical physicists even now use phenomenological formulas without a theoretical foundation, yet. However, in macroscopic electrodynamics, one can guess the structure of a formula in certain problems even before solving Maxwell's equations finally. In particular, we can often predict a formula in a complicated calculation of the dynamics of particle beams in accelerators on the basis of the following simple, but efficient, physical ideas [2]: Parameter dimension is important, the dimensionless parameters are of the same order, and after physical simplifications and transformations, the required formula possesses an algebraic structure.

These ideas are generally known, and used in the present paper to deduce simple algebraic formulas for the proton's and neutron's magnetic moments. Recall that the Balmer and Sommerfeld formulas (and not only they) for the spectrum of the hydrogen atom were derived similarly. The former accelerated the development of quantum mechanics, while the latter that of creation of relativistic quantum theory.

We make use of the mathematical constants $\sqrt{2},\sqrt{3},\sqrt{5},\pi$ and exp[1] that are frequently employed in quantum electrodynamics. Besides the above foundations, we require that the coefficients, which normally arise in quantization, should be integers or fractions of integers, what involved a very strong limitation in numerical analysis. These formulas are of exactly such structure in quantum mechanics and quantum electrodynamics to refer monographs. The Balmer and Sommerfeld formulas possess the same structure. It is not at all obvious that a complicated problem set in the present paper can be solved, however.

We regard a formula as compact if the number of the original constants used is less (or substantially less) than that of valid digits. E.g., one number, 2, is used to calculate the Bohr magneton, whereas 3 valid digits, 2.00, are obtained. That much accuracy permitted Bohr to accept the value 2 as valid. Three parameters, 2, $\boldsymbol{\alpha}$ and $\boldsymbol{\pi}$, are involved in the Schwinger formula for the first approximation to the anomalous magnetic moment of the electron in Bohr magneton units

$$\delta\mu_e \approx \alpha/2\pi \approx 1.1614\times 10^{-3}, \qquad (3)$$

while 5 valid digits after the decimal point are obtained [3, 4]. If we use a Taylor series in calculating exp[1] with 12 valid digits, then we obtain 15 addends. Taylor's algebraic formula is not compact by our definition, i.e. that cannot be accepted here.

However surprising it may seem, considering more than 50,000 formulas led us to simple algebraic formulas for the magnetic moments of the proton and neutron with 10 valid digits. Applying this method to the masses of the proton and muon also yielded simple relationships with 12 valid digits. Meanwhile, the electron's anomalous magnetic moment is connected to the moments in eq.(1 and 2) by a simple relationship which provides for a higher accuracy than the one resulted from quantum electrodynamics.

From the moments, we now turn to other variables. We use the *summary* moment

$$\mu_+ = |\mu_n| + \mu_p = 390.1818022^{-1} \quad (4)$$

and the *relative* moment

$$\mu_{\text{rel}} = \frac{|\mu_n|}{|\mu_n| + \mu_p} = 0.40652091103. \qquad (5)$$

It is just for these quantities our extremely simple formulas arise.

2. Relationship between summary magnetic moment μ_+ and mass of neutron

We use the experimental value for mass of the neutron

$$m_{\text{n}} = 1838.6836605(11) \quad (6)$$

in the numerical analysis of summary magnetic moment [1]. We should expect μ_+ to be approximately inversely proportional to the neutron mass. When we try to find the multiplier by numerical analysis, we arrive at the formula

$$\mu_+ = \frac{3\pi}{(2m_n + \sqrt{2}\exp[-5])}. \qquad (7)$$

In substituting experimental values of μ_+ and m_{n} for equation (7) to hold we should substitute 4.99998 for the 5, which confirms that the formula holds. If we calculate μ_+, then we get 10 valid digits, viz.,

$$(\mu_+)^{-1}_{cal} = (|\mu_n| + \mu_p)^{-1} = 390.181802182.$$

It follows from equation (7) that the factor of proportionality (without taking the small addend into account) equals 3π/2. We make use of 7 parameters in equation (7), whereas there are 3 more valid figures (probably, 5 more, as follows from Chapter 5). By the selected criterion, the formula is compact.

3. Dependence of neutron relative magnetic moment μ_{rel} on fine structure constant α

The fine structure constant α found experimentally as

$$\alpha^{-1}=137.035999074(44) \qquad (8)$$

(see [1]).

Following the above approach, using numerical analysis, we can arrive at the following formula for the neutron's relative magnetic moment

$$\mu_{\mathrm{rel}} = 5(8+\pi)\alpha + \frac{\alpha}{(12+\pi)m_n} \; . \qquad (9)$$

Substitution of the experimental values for μ_{rel} , m_{n} and α from eq. (5, 6 and 8), respectively, in the above parameters yields 11.9997 in the small latter addend instead of 12, and the formula structure is thus confirmed. Calculation of the neutron's relative moment

$$\mu_{rel\mathrm{cal}} = \frac{|\mu_n|}{|\mu_n| + \mu_p} = 0.406520911025$$

yields 10 valid figures. If we calculate α^{-1} in terms of μ_{rel} by equation (9) then that yields all the known digits, viz.,

$$\alpha^{-1}_{\mathrm{cal}} = 137.0359990741.$$

4. Relative magnetic moment μ_{rel} as function of number π^5

Moment μ_{rel} is approximately equal to 2/5. Note that the value of a correction to 2/5 is close to $2/\pi^5$**.** To make it precise, we use the difference between the neutron's and proton's masses

$$\Delta m = 2.53098805 \qquad (10)$$

(see [1]).

Then

$$1/(\mu_{rel}\tilde{}\,2/5)\tilde{}\;\Delta m\,\mu_{rel}/3\square\;\pi^5/2\,. \quad (11)$$

If we substitute experimental values for Δm and μ_{rel} on the left-hand side of formula (11), then for equation (11) to hold, we should substitute 2.00000003 for the 2 on the right-hand side.

If we calculate relative moment from equation (11) to the accuracy of measurements, then

$$\mu_{\mathrm{relcal}} = \frac{|\mu_n|}{|\mu_n| + \mu_p} = 0.406520910879\,.$$

5. Inverse corrections relationship

Numerical analysis shows that the difference between the inverse corrections to μ_{rel} and Δm with a good accuracy of 6 valid figures is equal to 8. More precisely, to the accuracy of measurement, the relationship

$$5/(\Delta m\text{-}5/2)\tilde{}\;1/(\mu_{rel}\text{-}2/5)\;\square\;8(\tilde{1}2\alpha^2/3) \qquad (12)$$

holds.

If we substitute experimental values for Δm and μ_{rel} in the above formula, then we obtain 137.034 for α^{-1}. If we calculate μ_{rel} in terms of α and Δm by equation (12) then

$$\mu_{\mathrm{rel}} = \frac{|\mu_n|}{|\mu_n| + \mu_p} = 0.406520911029\,,$$

while calculating Δm in terms of μ_{rel} and α yields all figures known experimentally, viz.,

$$\Delta m_{\mathrm{cal}} = 2.530988050003.$$

6. Relationship between nucleon summary moment μ_+ and electron's anomalous magnetic moment $\delta\mu_e$

The difference to a good accuracy between inverse values of μ_+ and $\delta\mu_e$ with $\sqrt{2}/\pi$ as a coefficient is equal to number 2. To make this relationship more precise, we introduce a small addend, which is linear in terms of the moments. As a result of numerical analysis, we get

$$\sqrt{2}/\pi\,\delta\tilde{\mu}_e 1/\mu_+ \,\square\,\tilde{}\,2 + \delta\mu_e + \mu_+/\sqrt{10}\,. \qquad (13)$$

Substituting experimental values of the electron's anomalous magnetic moment

$$\delta\mu_e = 1.15965218076(27)\times 10^{-3} \qquad (14)$$

(see [1]) and the summary moment μ_+ from datum (4) into equation (13) with a small addend on the right-hand side generates 10.0005 rather than 10. If we take μ_+ from Chapter 1 and calculate the electron's anomalous magnetic moment, then we get at least 14 valid digits after the decimal point, viz.,

$$\delta\mu_{ecal} = 1.1596521807579\times 10^{-3},$$

which is 4 digits better than what quantum electrodynamics offers [4].

Without the small addend on the right-hand side in equation (13), this formula yields 8 valid digits after the decimal point. The total one allowing us to predict 10 valid digits for summary moment

$$\mu_{+cal} = 390.181802181,$$

which well agrees with the result obtained in Chapter 2.

7. Ratio of mass of proton to that of electron

Equation (11) suggests that to construct the proton's dimensionless mass, we should make use of π^5 **.** In fact, the mass is approximately $6\pi^5$ with 5 valid digits. More precisely,

$$m_p = 6\pi^5 + \frac{1}{16\left[(\pi - 4/3) - 2(\pi/m_n)^2\right]}, \qquad (15)$$

where the 5 parameters (2, 3, 5, π and m_n) are involved.

If we find m_n from equation (15) by substituting the experimental value of

$$m_p = 1836.15267245(75) \qquad (16)$$

(see [1]) , then we get number1838. Calculation of m_p leads us to 12 valid digits, viz.,

$$m_{p\mathrm{cal}} = 1836.152672450232.$$

8. Neutron dimensionless mass

The main component of the formula is $2\pi(3\pi^4 + 1/\sqrt{6})$, which yields 7 valid digits. The following formula, making the latter quantity more precise, is simple too, viz.,

$$m_n = 2\pi\left\{3\pi^4 + \frac{1}{\sqrt{\left[6 - 1/(8\exp[1])^2\right]}}\right\} . \qquad (17)$$

Substituting the experimental value from data (6) in equation (17) yields 2.7183 rather than exp[1]. Calculation of mass

$$m_{n\mathrm{cal}} = 1838.683660508$$

yields all valid digits.

9. Mass difference Δm

Found formulas involve the mass difference. For them to involve none other than constants, we express Δm in terms of the fine structure constant α, assuming that Δm is proportional to α^2. We get a simple formula with 10 valid digits

$$\Delta m = \frac{5\pi^8\alpha^2}{[1 - \pi(\pi^2 + 1)\alpha^2]} \qquad (18)$$

by numerical analysis.

Substituting the value of α from equation (8) in formula (18) yields

$$\Delta m_{\mathrm{cal}} = 2.530988045475.$$

10. Relationship between neutron mass and fine-structure constant

Formula is very simple, viz.,

$$m_n - \frac{1}{\sqrt[4]{(m_p - 2 + 10\alpha)}} = 6\sqrt{5}\alpha^{-1} . \qquad (19)$$

Substituting the experimental values from eq. (4 and 16) in formula (19) with small addend under root yields 137.1 rather than α^{-1}. Calculation of mass m_n:

$$m_{ncal}=1838.683660499$$

yields all valid digits.

Conclusions

Applying the numerical analysis used to the main algebraic formulas of quantum electrodynamics [4] shows that any comparatively simple formula can be restored if sufficiently many valid digits and constants employed are known. If we apply the method to precisely measurable in experiments quantities, then we unexpectedly arrive at compact algebraic formulas for magnetic moments and mass ratios. These formulas are useful in studying various models in nuclear physics when high accuracy of calculations is required. Formula (7) for summary moment is extremely simple, which enables us to predict 2 more valid digits, taking into account the accuracy of the neutron's mass measurement.

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References

[1] 2010 CODATA. Values of the Fundamental Constants http://physics.nist.gov/cuu/Constants/index.html
[2] L.A. Yudin, S.P. Efimov, M.I. Kapchinsky, and I.L. Korenev, Phys. Plasmas, **3**, 42 (1996)
[3] J. Schwinger, Phys. Rev., **75**, 1912 (1949)
[4] V.B. Beresteskii, E.M. Lifshitz, and L.P. Pitaevskii, Quantum Electrodynamics, Pergamon, Oxford, p.652 (1982)
[5] T. Aoyama et al., Phys. Rev. Lett. **99**, 110406 (2007)