

# Understanding the discrete nature of angular momentum of electron in hydrogen atom with (3G, 2e) model of final unification

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**Abstract:** In the early publications, with reference to final unification, the authors suggested that, 1) There exists a strong interaction elementary charge of magnitude,  $e_s \sim 4.72058686E-19$  C. 2) Like quarks, the strong interaction elementary charge is experimentally undetectable and can be called as 'invisible elementary nuclear charge'. 3) There exists a gravitational constant associated with strong interaction,  $G_s \sim 3.329560807E28$  m<sup>3</sup>/kg/sec<sup>2</sup>. 4) There exists a gravitational constant associated with electromagnetic interaction,  $G_e \sim 2.374335472E37$  m<sup>3</sup>/kg/sec<sup>2</sup>. Based on these concepts, an attempt is made to understand the mystery of origin of 'discrete' angular momentum of electron in hydrogen atom. Proceeding further, estimated value of Newtonian gravitational constant is  $G_M \sim 6.67985603E-11$  m<sup>3</sup>/kg/sec<sup>2</sup>.

**Keywords:** 3 different gravitational constants, 2 different elementary charges, hydrogen atom, s - shell, final unification.

## 1. Introduction

### 1.1. About 'strong gravity' and 'strong nuclear charge'

Roberto Onofrio says: "It is worth to point out that, with different motivations, the concept of 'strong gravity' has appeared from time to time in the literature, especially in connection with the possibility that gravity plays a role in the confinement of quarks inside hadrons through black-hole analogies, although not within the framework of considering weak interactions as derivable from gravity at short length scale" [1,2].

According to Roberto Onofrio [1], weak interactions are peculiar manifestations of quantum gravity at the Fermi scale, and that the Fermi coupling constant is related to the Newtonian constant of gravitation. In his opinion. at atto-meter scale, Newtonian gravitational constant seems to reach a magnitude of  $8.205 \times 10^{22}$  m<sup>3</sup>kg<sup>-1</sup>sec<sup>-2</sup>. In this context, one can see plenty of papers on 'strong gravity' in physics literature [3-19]. It may be noted that, till date, 'strong gravity' is a non-mainstream theoretical approach to Color confinement/particle confinement having both a cosmological scale and a particle scale gravity. In between ~ (1960 to 2000), it was taken up as an alternative to the then young QCD theory by several theorists, including Abdus Salam

[3]. Very interesting point to be noted is that, Abdus Salam showed that the 'particle level gravity approach' can produce confinement and asymptotic freedom while not requiring a force behavior differing from an inverse-square law, as does QCD. C. Sivaram published a review of this [4].

Qualitatively and quantitatively, references [1-20] strongly suggest the possible existence of 'Newtonian (like) gravitational constant with very large magnitude' in nuclear and particle physics. Based on this concept and in pursuit of bridging the gap in between 'General theory of relativity' and 'Quantum field theory' [21-24], in the recent publications [25-30], the authors suggested and validated the role of two gravitational constants associated with strong and electromagnetic interactions.

Proceeding further, the authors also suggested and validated the role of a new elementary charge associated with nuclear physics and currently believed strong coupling constant [31,32]. This new elementary charge can be compared with the historical strong interaction elementary charge. It may be noted that, in nuclear physics literature, starting from 1950's scientists supposed the existence of a new type of 'charge' associated with strong interaction. In analogy with electromagnetic interaction strength  $\alpha \cong e^2/4\pi$ , quantum chromo dynamics [33] presumes the strong interaction strength as  $\alpha_s \cong g_s^2/4\pi$ . Considering many body

nuclear system, strong elementary charge was assumed to be playing a key role [34-39]. In this connection, with reference the old historical idea of 'strong nuclear chare', in this paper, the authors made an attempt in fixing and extending the applications of 'strong nuclear charge' starting from the 'strong coupling constant' to the observable nuclear properties like 'magnetic moments' of nucleons, 'nuclear binding energy' and 'nuclear stability'.

## 1.2. About 'unification of quantum mechanics' and 'general theory of relativity'

Even though 'String theory' and 'Quantum gravity' models [40,41] are having a strong mathematical back ground and sound physical basis, both the models are failing in developing a 'workable' model of final unification. In this context, at fundamental level, starting from sub-nuclear physics to low energy (observable) nuclear physics, along with the proposed 'new nuclear elementary charge' proposed two gravitational constants assumed to be associated with electron and proton seem to play a vital role in understanding the basics of final unification. In an integrated approach the authors also showed that, 'quantum of angular momentum' is a characteristic result of the combined effects of gravitational constants associated with proton and electron. Proceeding further, the authors discovered simple relations that seem to be connected with the three gravitational constants i.e, Newtonian gravitational constant and the proposed two gravitational constants assumed to be associated with proton and electron.

## 1.3. Key topics of this paper

In this paper,

1. The authors revised the third assumption and compiled important characteristic relations pertaining to 'final unification'.
2. Made an attempt to understand the mystery of discrete nature of revolving electron's discrete angular momentum.
3. Proposed three simple semi empirical relations for estimating the Newtonian gravitational constant.

## 2. Three basic assumptions of final unification

In the recent publications [25-30] the authors proposed and established three assumptions. Here, in this paper the authors revised the third assumption for better understanding.

**Assumption-1:** Magnitude of the gravitational constant associated with the electromagnetic interaction is,  $G_e \cong 2.374335472 \times 10^{37} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}$ .

**Assumption-2:** Magnitude of the gravitational constant associated with the strong interaction is,  $G_s \cong 3.329560807 \times 10^{28} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2}$ .

**(Revised) Assumption-3:** There exists a strong elementary charge,  $e_s \cong 4.72058686 \times 10^{-19} \text{ C}$ . Like quarks, the strong interaction elementary charge is experimentally undetectable and can also be called as 'invisible elementary nuclear charge'.

## 3. Important and characteristic unified results

Considering the following semi empirical results one can understand and validate the role of the proposed three assumptions.

$$\text{Here } \begin{cases} e \cong 1.602\ 176\ 565(35) \times 10^{-19} \text{ C}, \\ \epsilon_0 \cong 8.854\ 187\ 817 \times 10^{-19} \text{ F/m} \\ m_n \cong 1.674\ 927\ 471(21) \times 10^{-27} \text{ kg}, \\ m_p \cong 1.672\ 621\ 777(74) \times 10^{-27} \text{ kg} \\ m_e \cong 9.109\ 382\ 91(40) \times 10^{-31} \text{ kg}, \\ \hbar \cong 1.054\ 571\ 726(47) \times 10^{-34} \text{ J.sec.} \\ \alpha \cong 7.297\ 352\ 5698(24) \times 10^{-3} \end{cases}$$

$$\text{and } \begin{cases} G_s \cong 3.329560807 \times 10^{28} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \\ G_e \cong 2.374335472 \times 10^{37} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-2} \\ e_s \cong 4.72058686 \times 10^{-19} \text{ C} \end{cases}$$

### 1) Nuclear charge radius:

$$R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.239291 \times 10^{-15} \text{ m} \quad (1)$$

### 2) Root mean square radius of proton:

$$R_p \cong \frac{\sqrt{2}G_s m_p}{c^2} \cong 0.876311 \times 10^{-15} \text{ m} \quad (2)$$

### 3) Fermi's weak coupling constant:

$$\text{If } \frac{G_s m_p^2}{R_0^2} \cong \frac{c^4}{4G_s},$$

$$F_W \cong \left( \frac{e_e}{e_s} \right) \left[ \frac{(G_s m_p^2)(G_s m_e^2)}{(c^4/4G_s)} \right] \quad (3)$$

$$\cong 1.44021 \times 10^{-62} \text{ J.m}^4$$

4) **Bohr radius of electron:**

$$a_0 \cong \left( \frac{4\pi\epsilon_0 G_e m_e^2}{e_e^2} \right) \left( \frac{G_s m_p}{c^2} \right) \quad (4)$$

This is one crystal clear result of the proposed ( $G_e, G_s$ ). See section-3 for its potential application.

5) **Fine structure ratio:**

$$\alpha \cong \frac{e_e^2}{4\pi\epsilon_0 \hbar c} \cong \frac{e_s e_e}{4\pi\epsilon_0 G_s m_p^2} \quad (5)$$

6) **Strong interaction strength:**

$$\beta \cong \frac{e_s^2}{4\pi\epsilon_0 \hbar c} \cong \left( \frac{e_s}{e_e} \right) \frac{e_s^2}{4\pi\epsilon_0 G_s m_p^2} \quad (6)$$

$$\cong 0.06334853354$$

7) **Ratio of Strong and electromagnetic interaction strengths:**

$$\frac{\alpha}{\beta} \cong \left( \frac{e_e}{e_s} \right)^2 \cong 0.1151937095 \quad (7)$$

Here, very interesting point to be noted that, ( $\alpha/\beta$ ) seems to be matching with the currently believed 'strong coupling constant'  $\alpha_s$  [9,10]. Geometric mean strength of strong and electromagnetic interactions i.e.  $\sqrt{\alpha\beta}$  seems to play a crucial role in nuclear binding energy scheme [27]. See the following relation (8).

8) **Nuclear binding energy close to stable atomic nuclides' beginning range:**

$$BE \approx \left( Z - 2 + \sqrt{\frac{Z}{30}} \right) (\sqrt{\alpha\beta} - \alpha\beta) (m_p c^2)$$

$$\approx \left( Z - 2 + \sqrt{\frac{Z}{30}} \right) 19.74 \text{ MeV} \quad (8)$$

where  $Z \geq 5$

Note that, according to Fermi gas model of nucleus [27], mean kinetic energy of nucleon is roughly 20 MeV and can be fitted with  $\sqrt{\alpha\beta} (m_p c^2) \cong 20.173 \text{ MeV}$ .

9) **Ratio of rest mass of proton and electron:**

$$\left( \frac{m_p}{m_e} \right) \cong \left( \frac{4\pi\epsilon_0 G_e m_e^2}{e_e^2} \right) \left/ \left( \frac{4\pi\epsilon_0 G_s m_p^2}{e_s^2} \right) \right. \quad (9A)$$

$$\rightarrow \left( \frac{m_p}{m_e} \right) \cong \left( \frac{e_s^2 G_e}{e_e^2 G_s} \right)^{1/3} \cong \left( \frac{\beta G_e}{\alpha G_s} \right)^{1/3}$$

If is  $G_N$  the Newtonian gravitational constant, it is noticed that,

$$\left( \frac{m_p}{m_e} \right) \cong \sqrt{\frac{e_e}{e_s}} \sqrt{\frac{G_s}{G_N^{1/3} G_e^{2/3}}} \cong \sqrt{\frac{e_e G_s}{e_s G_N^{1/3} G_e^{2/3}}}$$

$$\Rightarrow G_N \cong \left( \frac{e_e}{e_s} \right)^7 \left( \frac{G_s^5}{G_e^4} \right) \quad (9B)$$

$$\cong 6.679856043 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$$

See section-4 for further information.

10) **Specific charge ratio of proton and electron:**

$$k \cong \left( \frac{G_s m_p m_e}{\hbar c} \right) \cong \left( \frac{\hbar c}{G_e m_e^2} \right) \cong \left[ \left( \frac{e_s}{m_p} \right) \left/ \left( \frac{e_e}{m_e} \right) \right. \right]$$

$$\cong \frac{\text{specific charge of proton associated with } e_s}{\text{specific charge of electron associated with } e_e} \quad (10)$$

$$\cong 1.604637101 \times 10^{-3}$$

Note that, this ratio seems to play a key role in understanding 'electronic stability' in hydrogen atom and 'proton-neutron stability' in nuclear physics and casts doubt on the independent existence of 'quantum constants'.

11) **Reduced Planck's constant:**

$$\hbar \cong \left[ \left( \frac{e_s}{m_p} \right) \left/ \left( \frac{e_e}{m_e} \right) \right. \right] \left( \frac{G_e m_e^2}{c} \right)$$

$$\cong \left( \frac{e_s}{e_e} \right) \left( \frac{m_e}{m_p} \right) \left( \frac{G_e m_e^2}{c} \right) \cong \left( \frac{e_s}{e_e} \right) \left( \frac{G_s m_p^2}{c} \right) \quad (11A)$$

$$\cong \left( \frac{m_e}{m_p} \right)^{1/2} \left[ \frac{\sqrt{(G_s m_p^2)(G_e m_e^2)}}{c} \right]$$

Alternatively, it is also noticed that,

$$\left. \begin{aligned} \hbar &\equiv \left( \frac{e_e}{e_s} \right)^2 \left( \frac{G_s}{G_N^{1/3} G_e^{2/3}} \right) \left( \frac{G_s m_e^2}{c} \right) \\ \alpha &\equiv \left( \frac{G_N^{1/3} G_e^{2/3}}{G_s} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 G_s m_e^2} \right) \end{aligned} \right\} \quad (11B)$$

**12) Proton-neutron beta stability line:**

$$\left. \begin{aligned} A_s &\cong 2Z + \left[ \left( \frac{e_s}{m_p} \right) / \left( \frac{e_e}{m_e} \right) \right] (2Z)^2 \\ &\cong 2Z + \left( \frac{e_s m_e}{e_e m_p} \right) (2Z)^2 \\ &\cong 2Z + \left( \frac{G_s m_p m_e}{\hbar c} \right) (2Z)^2 \\ &\cong 2Z + (0.0064185Z)^2 \end{aligned} \right\} \quad (12)$$

where  $A_s$  is the stable mass number of  $Z$ .

**13) Magnetic moment of electron:**

$$\mu_p \cong \frac{e_e \hbar}{2m_e} \cong \left( \frac{e_e}{e_s} \right)^2 \left( \frac{G_s}{G_N^{1/3} G_e^{2/3}} \right) \left( \frac{G_s m_e e_e}{2c} \right) \quad (13)$$

**14) Magnetic moment of muon:**

$$\mu_\mu \cong \frac{e_e \hbar}{2m_\mu} \cong \left( \frac{e_e}{e_s} \right)^2 \left( \frac{G_s}{G_N^{1/3} G_e^{2/3}} \right) \left( \frac{m_e}{m_\mu} \right) \left( \frac{G_s m_e e_e}{2c} \right) \quad (14)$$

**15) Magnetic moment of tau:**

$$\mu_\tau \cong \frac{e_e \hbar}{2m_\tau} \cong \left( \frac{e_e}{e_s} \right)^2 \left( \frac{G_s}{G_N^{1/3} G_e^{2/3}} \right) \left( \frac{m_e}{m_\tau} \right) \left( \frac{G_s m_e e_e}{2c} \right) \quad (15)$$

**16) Magnetic moment of proton:**

$$\mu_p \cong \frac{e_s \hbar}{2m_p} \cong \left( \frac{e_e}{e_s} \right)^2 \left( \frac{G_s}{G_N^{1/3} G_e^{2/3}} \right) \left( \frac{m_e}{m_p} \right) \left( \frac{G_s m_e e_s}{2c} \right) \quad (16)$$

**17) Magnetic moment of neutron:**

$$\left. \begin{aligned} \mu_n &\cong \left( \frac{e_s \hbar}{2m_n} - \frac{e_e \hbar}{2m_n} \right) \cong \frac{\hbar}{2m_n} (e_s - e_e) \\ &\cong \left( \frac{e_e}{e_s} \right)^2 \left( \frac{G_s}{G_N^{1/3} G_e^{2/3}} \right) \left( \frac{m_e}{m_n} \right) \left[ \frac{G_s m_e (e_s - e_e)}{2c} \right] \end{aligned} \right\} \quad (17)$$

**3. Understanding the Mystery of Quantum Nature of Electron in Hydrogen Atom**

Considering relations (1) to (17), the authors would like to stress the following facts.

- A) Along with the new strong elementary charge, within the atomic medium there exit two different gravitational constants and their existence is real, not virtual.
- B) Considering ( $G_s$  and  $G_e$ ) magnitudes of quantum constants like ‘basic unit of angular momentum’, ‘basic unit of electron’s distance’ etc. can be fitted and understood.
- C) It may be noted that, according to Bohr’s theory of hydrogen atom [42,43], number of electrons that can be accommodated in any principal quantum shell is  $2n^2$ . Based on this idea, it is possible to assume that, probability of finding any one electron is  $\left( \frac{1}{2n^2} \right)$ . It can be obtained in the following way.
- D) Out of  $2n^2$  electrons, number of electrons that can be accommodated in  $s$  shell is 2. If one is willing to consider  $s$  shell as a basic entity in such a way that,  $p$  shell constitutes  $3s$  shells,  $d$  shell constitutes  $5s$  shells,  $f$  shell constitutes  $7s$  shells etc, then,  $n^2$  can be considered as a representation of total number of  $s$  shells that can be accommodated in any principal quantum shell.
- E) Notation point of view, it can be assigned for  $p$  shell:  $ps1, ps2, ps3$  and for  $d$  shell:  $ds1, ds2, ds3, ds4, ds5$  etc. Transition of electron from  $2^{nd}$  orbit  $p$  shell to 1st orbit  $s$  shell can be expressed as:  $2ps1$  to  $1s, 2ps2$  to  $1s, 2ps3$  to  $1s$ . Thinking in this way different transition levels can be expected. With reference to  $p$  shell, 3 different spectral lines, with reference to  $d$  shell, 5 different spectral lines can be expected. Similarly with reference to  $f$  shell, 7 different spectral lines can be expected.
- F) If so, it is also possible to assume that, probability of finding any one  $s$  shell is  $\left( \frac{1}{n^2} \right)$ .

Based on this proposal, from relation (4), discrete potential energy of  $s$  shell in hydrogen atom can be expressed as follows.

$$E_{pot} \cong - \left( \frac{1}{n^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2 c^2}{4\pi\epsilon_0 G_s m_p} \right) \quad (18)$$

where  $\left(\frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2}\right)$  represents a force ratio and  $n^2$  represents the total number of  $s$  shells corresponding to  $n^{\text{th}}$  principal quantum shell. Thinking in this way, orbiting radius of  $n^2$  number of  $s$  shells can be expressed as,

$$a_n \cong n^2 \left\{ \left( \frac{4\pi\epsilon_0 G_e m_e^2}{e_e^2} \right) \left( \frac{G_s m_p}{c^2} \right) \right\} \quad (19)$$

$$\cong n^2 \left\{ \left( \frac{m_p}{m_e} \right) \left( \frac{4\pi\epsilon_0 G_s m_p^2}{e_s^2} \right) \left( \frac{G_s m_p}{c^2} \right) \right\}$$

Clearly speaking,  $a_n$  represents the orbiting radius of  $n^2$   $s$  shells. In this way, the long standing concept of 1:4:9:16 etc. can be understood in a more meaningful approach.

Now,  $s$  shell's discrete kinetic energy can be expressed as follows.

$$E_{kin} \cong \frac{1}{2} |E_{pot}|$$

$$\cong \left( \frac{1}{2n^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2 c^2}{4\pi\epsilon_0 G_s m_p} \right) \quad (20)$$

Discrete total energy of one  $s$  shell can be expressed as follows.

$$E_{tot} \cong - \left( \frac{1}{2n^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2 c^2}{4\pi\epsilon_0 G_s m_p} \right)$$

$$\cong - \left( \frac{1}{n^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2 c^2}{8\pi\epsilon_0 G_s m_p} \right)$$

$$\cong - \left( \frac{1}{n^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 (2G_s m_p / c^2)} \right)$$

$$\cong - \left( \frac{1}{n^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 R_0} \right)$$

$$\left. \begin{array}{l} \text{where } n = 1, 2, 3, \dots \text{ and} \\ R_0 \cong (2G_s m_p / c^2) \cong 1.239291 \text{ fm} \end{array} \right\} \quad (21)$$

Here important points to be noted are:

1.  $\left(\frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2}\right)$  is the ratio of electromagnetic and gravitational force of electron where the operating gravitational constant is  $G_e$  not  $G_N$ .
2.  $\left(\frac{e_e^2}{4\pi\epsilon_0 R_0}\right) \approx 1.16 \text{ MeV}$  is the currently believed characteristic nuclear coulombic potential.
3. Ratio of total ground state energy of electron in hydrogen atom and characteristic nuclear coulombic potential is equal to ratio of electromagnetic and gravitational force of electron where the operating gravitational constant is  $G_e$ .

### 3.1 Understanding the origin of 'quantum of angular momentum' in hydrogen atom

Here it may be noted that, in the hydrogen atom, there exists only one electron. Hence relation (26) can be considered as a representation of the total energy of electron. Comparing this relation (21) with Bohr's theory of hydrogen atom, relation (11) can be obtained with the following relation.

$$\left( \frac{e_e^4}{32\pi^2 \epsilon_0^2 n^2 \hbar^2} \right)^2 \cong \left( \frac{1}{2n^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2 c^2}{4\pi\epsilon_0 G_s m_p} \right) \quad (22)$$

$$\hbar \cong \left( \frac{m_e}{m_p} \right)^{\frac{1}{2}} \left[ \frac{\sqrt{(G_s m_p^2)(G_e m_e^2)}}{c} \right] \quad (22)$$

From relation (9),

$$\frac{G_s}{G_e} \cong \frac{e_s^2}{e_e^2} \times \frac{m_e^3}{m_p^3} \quad (23)$$

Following this relation (23), relation (22) can be written into two different forms as expressed in relation (11).

### 3.2 Understanding the integral nature of 'quantum of angular momentum' in hydrogen atom

If one is willing to consider the following three points, it is possible to understand the integral nature of electron's angular momentum,

1. Within the atom, electronic arrangement is 'systematic'.

2. In  $n^{\text{th}}$  principal quantum shell, there is a scope for the existence of  $n^2$  number of (currently believed) s-shells.
3. 3 number of s-shells can be collectively called as one 'p-shell'. Similarly 5 number of s-shells can be collectively called as one 'd-shell' and so on.

Now the famous expression for integral nature of angular momentum can be expressed as:

$$\hbar_n \cong n \left\{ \sqrt{\frac{m_e}{m_p} \frac{\sqrt{(G_s m_p^2)(G_e m_e^2)}}{c}} \right\} \quad (24)$$

Here,  $n \cong \sqrt{n^2}$  represents the number of s shells. In hydrogen atom, as there exists only one s-shell and one electron, it appears from Bohr's theory of hydrogen atom, that – revolving electron's angular momentum is  $n\hbar$  and distance is  $n^2 a_0$ . This is the key point to be noted here.

The emitted energy can be expressed as follows.

$$\left. \begin{aligned} E_{emis} &\cong \left\{ \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2 c^2}{8\pi\epsilon_0 G_s m_p} \right) \right\} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \\ &\cong \left\{ \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 (2G_s m_p / c^2)} \right) \right\} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \\ &\cong \left\{ \left( \frac{e_e^2}{4\pi\epsilon_0 G_e m_e^2} \right) \left( \frac{e_e^2}{4\pi\epsilon_0 R_0} \right) \right\} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \end{aligned} \right\} \quad (25)$$

where  $n_2^2 > n_1^2$  and  $R_0 \cong (2G_s m_p / c^2) \cong 1.239291 \text{ fm}$

#### 4. To fit the Newtonian gravitational constant

It may be noted that, coupling Newtonian gravitational constant  $G_N$  with elementary physical constants is really a challenging issue and demands sound physical reasoning. In the earlier publications [44-50] and references therein, the authors proposed interesting semi empirical relations. With the proposed assumptions, it is noticed that, proton-electron mass ratio, elementary charge ratio and ratio of any two gravitational constants etc. seem to play a key role in this new approach. The authors are on the way to understand the 'back ground physics' of these relations. With further research, in near future, exact

unified relations can be developed and absolute value of  $G_N$  can be estimated [51-56].

With reference to Proton-Electron mass ratio and proposed assumptions, it is noticed that,

$$\sqrt{\frac{e_s}{e_e} \left( \frac{m_p}{m_e} \right)} \cong \sqrt{\frac{G_s}{G_N^{1/3} G_e^{2/3}}} \quad (26)$$

With reference to Planck mass  $M_{pl} \cong \sqrt{\hbar c / G_N}$  and proposed assumptions, it is noticed that,

$$\frac{\sqrt{M_{pl} m_e}}{m_p} \cong \left( \frac{G_e}{G_N} \right)^{1/6} \quad (27)$$

Interesting observation is that,

$$\sqrt{(G_e / G_N)} \approx 5.964622 \times 10^{23} \quad (28)$$

This number is very close to the Avogadro number [27-34]. Alternative expression can also be expressed as follows.

$$\left( \frac{\sqrt{M_{pl} m_p}}{m_e} \right)^3 \cong \sqrt{\frac{e_e}{e_s} \left( \frac{G_s}{G_N} \right)} \quad (29)$$

Thus,

$$\begin{aligned} G_N &\cong \left( \frac{e_e}{e_s} \right)^7 \left( \frac{G_s^5}{G_e^4} \right) \\ &\cong 6.679856043 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2} \end{aligned} \quad (30)$$

This estimated value of  $G_N$  can be compared with the experimental values [51-56]. It may be noted that, as gravity is much weaker than other fundamental forces and an experimental apparatus cannot be separated from the gravitational influence of other bodies,  $G_N$  is quite difficult to measure. So far, no standard model could couple gravity with other fundamental forces and hence it does not appear possible to calculate the value of  $G_N$  directly from other (accurate) microscopic physical constants. In addition, published values of  $G_N$  have varied rather broadly, and some recent measurements of high precision are, in fact, mutually exclusive. In 2007, Fixler et al [52] described a new measurement of the gravitational constant by 'atom interferometry', reporting a value of  $G_N \cong 6.693(34) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$ . An improved

cold ‘atom measurement’ by Rosi et al [53] was published in 2014 and reported a value of  $G_N \cong 6.67191(99) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$ . Most recent (CODATA: 2014) recommended value of  $G_N$  is  $6.67408(31) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$ . In this context, the authors would like to stress the fact that, fitting the value of  $G_N$  with ‘unification methodology’ is quite different from the existing experimental methods of  $G_N$  and seems to be promising and versatile.

## 5. Conclusion

Now a days, ‘String theory’ [40,41] is being believed as a promising candidate for a ‘quantum theory of gravity’. It was first studied in the late 1960s as a theory of the strong nuclear force. Even though it is having a strong mathematical background and sound physical footing, so far, string theory could not provide any clue for understanding the observed elementary particle mass spectrum and atomic and nuclear structures in terms of gravity. In this context, qualitatively and quantitatively, by considering the proposed concepts and relations, the authors would like to stress the following points.

- A) The proposed three assumptions can be given priority at fundamental level.
- B) Characteristic quantum physical constants that are believed to be ‘unique physical constants’ are compound in nature and hence can be considered as ‘secondary physical constants’.
- C) Discrete nature of orbiting electron can be better understood with ‘systematic arrangement’ of  $n^2$  number of s-shells.
- D) If one is willing to explore the possibility of incorporating the proposed assumptions either in ‘String theory’ models or in ‘Quantum gravity’ models or ‘Strong gravity’ models, certainly, background physics assumed to be connected with proposed semi empirical relations can be understood and a ‘practical’ model of “everything” can be developed.

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